Principles of Economics Exercises and Solutions

Exercise 1: Advanced Supply and Demand Analysis

Problem:

1. Consider a market where the demand function is $Q_d = 50 - 2P$ and the supply function is $Q_s = 10 + 3P$.
   (a) Find the equilibrium price and quantity.
   (b) Calculate the consumer surplus and producer surplus at equilibrium.
2. Suppose the government imposes a price ceiling of \$5.
   (a) Determine the quantity demanded and supplied at this price.
   (b) Analyze the resulting shortage or surplus and its implications for the market.

Solution:

1. **Equilibrium Price and Quantity:**

$$\begin{aligned} Q_d &= Q_s \\ 50 - 2P &= 10 + 3P \\ 40 &= 5P \\ P &= 8 \end{aligned}$$

   Substituting $P = 8$ into either equation to find Q:

$$Q = 50 - 2(8) = 34$$

   Equilibrium price is \$8, and equilibrium quantity is 34 units.

2. **Consumer Surplus (CS) and Producer Surplus (PS):**

$$\begin{aligned} \text{CS} &= \frac{1}{2} \times (25 - 8) \times 34 = \frac{1}{2} \times 17 \times 34 = 289 \\ \text{PS} &= \frac{1}{2} \times (8 - 2) \times 34 = \frac{1}{2} \times 6 \times 34 = 102 \end{aligned}$$

3. **Price Ceiling of $5:**

$$\begin{aligned} \text{Quantity demanded:} \quad & Q_d = 50 - 2(5) = 40 \\ \text{Quantity supplied:} \quad & Q_s = 10 + 3(5) = 25 \\ \text{Shortage:} \quad & Q_d - Q_s = 40 - 25 = 15 \end{aligned}$$

Implications:

- There will be a shortage of 15 units.
- Consumers will face difficulties finding the product.
- Black markets might emerge.

Exercise 2: Elasticity and Revenue

Problem:

1. Given the demand function $Q_d = 100 - 5P$, calculate the price elasticity of demand when the price is $10.
2. Analyze how a change in price from $10 to $8 will affect total revenue.
3. Discuss the implications of elasticity for a firm's pricing strategy when demand is elastic versus inelastic.

Solution:

1. **Price Elasticity of Demand:**

$$\begin{aligned} Q_d &= 100 - 5(10) = 50 \\ \text{Elasticity} &= \frac{dQ}{dP} \times \frac{P}{Q} \\ \frac{dQ}{dP} &= -5 \\ \text{Elasticity} &= -5 \times \frac{10}{50} = -1 \end{aligned}$$

Elasticity at $10 is -1.

2. **Change in Total Revenue:**

$$\begin{aligned} \text{Original Revenue at \$10:} \quad & TR = P \times Q = 10 \times 50 = 500 \\ \text{New Quantity at \$8:} \quad & Q_d = 100 - 5(8) = 60 \\ \text{New Revenue at \$8:} \quad & TR = 8 \times 60 = 480 \end{aligned}$$

Revenue decreases from $500 to $480.

3. **Implications for Pricing Strategy:**

- When demand is elastic (elasticity < -1), lowering prices increases total revenue.
- When demand is inelastic (elasticity > -1), raising prices increases total revenue.
- Firms must understand elasticity to set optimal prices.

Exercise 3: Cost Structures and Profit Maximization

Problem:

1. Assume a firm's total cost (TC) function is $TC = 100 + 20Q + 5Q^2$ and its total revenue (TR) function is $TR = 150Q - 3Q^2$.
   (a) Calculate the profit-maximizing output level.
   (b) Determine the price at this output level.
   (c) Calculate the maximum profit.
2. Analyze how changes in fixed costs affect the profit-maximizing level of output.

Solution:

1. **Profit Maximization:**

$$\begin{aligned}\pi &= TR - TC \\ \pi &= (150Q - 3Q^2) - (100 + 20Q + 5Q^2) \\ \pi &= 130Q - 8Q^2 - 100\end{aligned}$$

   First-order condition $\frac{d\pi}{dQ} = 0$:

$$\begin{aligned}\frac{d\pi}{dQ} &= 130 - 16Q = 0 \\ Q &= 8.125\end{aligned}$$

2. **Price at this output level:**

$$\begin{aligned}P &= 150 - 3Q \\ P &= 150 - 3(8.125) = 125.625\end{aligned}$$

3. **Maximum Profit:**

$$\begin{aligned}\pi &= 130(8.125) - 8(8.125)^2 - 100 \\ \pi &= 1056.25 - 528.125 - 100 = 428.125\end{aligned}$$

   Maximum profit is $428.125.

4. **Effect of Changes in Fixed Costs:**
   - Changes in fixed costs (e.g., ΔFC) do not affect the profit-maximizing output level as they do not change marginal cost.
   - Fixed costs affect total profit but not the output level.

Exercise 4: Market Structures and Strategic Behavior

Problem:

1. Consider two firms in a Cournot duopoly where each firm's cost function is $C(Q) = 20Q$ and the market demand function is $P = 100 - Q$.
   (a) Find the reaction functions for each firm.
   (b) Determine the Nash equilibrium quantities for both firms.
   (c) Calculate the equilibrium price and each firm's profit.
2. Discuss how the outcome would change if the firms were in a Bertrand competition instead of Cournot.

Solution:

1. **Cournot Duopoly:**

$$\begin{aligned} Q &= Q_1 + Q_2 \\ P &= 100 - (Q_1 + Q_2) \end{aligned}$$

Firm 1's profit:

$$\begin{aligned} \pi_1 &= (100 - Q_1 - Q_2)Q_1 - 20Q_1 \\ \frac{d\pi_1}{dQ_1} &= 100 - 2Q_1 - Q_2 - 20 = 0 \\ 80 - 2Q_1 - Q_2 &= 0 \\ Q_1 &= \frac{80 - Q_2}{2} \end{aligned}$$

Firm 2's reaction function is similar:

$$Q_2 = \frac{80 - Q_1}{2}$$

Solving the system of equations:

$$
\begin{aligned}
Q_1 &= \frac{80 - \frac{80-Q_1}{2}}{2} \\
Q_1 &= \frac{80 - 40 + 0.5Q_1}{2} \\
Q_1 &= \frac{40 + 0.5Q_1}{2} \\
Q_1 &= 26.67 \\
Q_2 &= 26.67
\end{aligned}
$$

Nash equilibrium quantities: $Q_1 = Q_2 = 26.67$**.**

2. **Equilibrium Price and Profit:**

$$
\begin{aligned}
\text{Total quantity:} \quad & Q = 53.34 \\
\text{Price:} \quad & P = 100 - 53.34 = 46.66 \\
\text{Profit:} \quad & \pi_1 = \pi_2 = (46.66 - 20) \times 26.67 = 707.11
\end{aligned}
$$

3. **Bertrand Competition:**

$$
\begin{aligned}
\text{Price:} \quad & P = MC = 20 \\
\text{Quantity:} \quad & Q = 100 - 20 = 80 \\
\text{Each firm produces half:} \quad & Q_1 = Q_2 = 40
\end{aligned}
$$

Profit is zero for each firm as $P = MC$**.**

Exercise 5: Economic Growth and Productivity

Problem:

1. Using the Solow growth model, assume an economy with the production function $Y = K^{0.3}L^{0.7}$.
   (a) Derive the steady-state level of capital per worker (k^*).
   (b) Analyze the impact of an increase in the savings rate on the steady-state capital and output per worker.
2. Discuss the role of technological progress in achieving sustained economic growth beyond the steady state.

Solution:

1. **Steady-State Capital per Worker:**

$$
\begin{aligned}
y &= k^{0.3} \\
sy &= sk^{0.3} \\
\delta k + nk &= sk^{0.3}
\end{aligned}
$$

Steady-state:

$$k^* = \left(\frac{s}{\delta + n}\right)^{\frac{1}{1-0.3}}$$
$$k^* = s^{\frac{1}{0.7}} \quad \text{(assuming } \delta + n = 1\text{)}$$

2. **Impact of Increased Savings Rate:**

$$k^* = \left(\frac{s}{\delta + n}\right)^{\frac{1}{0.7}}$$
$$k^* = s^{\frac{1}{0.7}}$$
$$y^* = (k^*)^{0.3}$$

3. **Technological Progress:**
   - Shifts production function up.
   - Increases output per worker (y) beyond steady state.
   - Enables sustained growth as A (technology) improves.

Exercise 6: Fiscal and Monetary Policy Effects

Problem:

1. Assume the economy is described by the IS-LM model with the IS curve given by $Y = C(Y - T) + I(r) + G$ and the LM curve given by $M/P = L(Y, r)$.
   (a) Derive the IS and LM curves and determine the equilibrium levels of income (Y) and interest rate (r).
   (b) Analyze the impact of a fiscal expansion (increase in G) on the equilibrium income and interest rate.
2. Discuss the short-term and long-term effects of a monetary expansion on output and price levels using the AD-AS model.

Solution:

1. **IS and LM Curves:**

$$\text{IS curve:} \quad Y = C(Y - T) + I(r) + G$$
$$\text{LM curve:} \quad \frac{M}{P} = L(Y, r)$$

2. **Fiscal Expansion:**

- Increase in G shifts IS curve right.
- Higher Y and r in short term.
- Crowding out effect if r increases significantly.

3. **Monetary Expansion:**
   - Short-term: AD shifts right, increasing Y and P.
   - Long-term: Y returns to natural level, P higher permanently.
   - Neutrality of money in long run, focus on inflation.

Exercise 7: International Trade and Policy

Problem:

1. Consider a two-country, two-good model where Country A and Country B produce goods X and Y. Given the following production possibilities:
   - Country A: 100X or 50Y
   - Country B: 80X or 40Y

   (a) Determine the comparative advantage for each country.

   (b) Explain the potential gains from trade for both countries.

2. Analyze the impact of imposing tariffs on good Y in Country A.

   (a) Discuss the welfare effects on consumers, producers, and the government.

   (b) Consider the possible retaliation by Country B and its implications for global trade relations.

Solution:

1. **Comparative Advantage:**
   - Opportunity cost in Country A: 1Y = 2X.
   - Opportunity cost in Country B: 1Y = 2X.
   - Both have same comparative advantage, specialize based on absolute advantage.

2. **Gains from Trade:**
   - Both specialize in producing more efficiently.
   - Trade increases total output and consumption.

3. **Tariffs on Good Y:**

- Increase domestic price, reduce imports.
- Protect domestic producers but hurt consumers.
- Government gains tariff revenue.
- Possible retaliation by Country B, reducing overall trade.

Exercise 8: Labor Market Dynamics

Problem:

1. Assume a labor market described by the following supply and demand functions:
   - Labor demand: $L_d = 50 - 2w$
   - Labor supply: $L_s = 10 + 3w$

   (a) Determine the equilibrium wage and employment level.

2. Analyze the effects of introducing a minimum wage above the equilibrium wage.

   (a) Calculate the resulting surplus or shortage of labor.

   (b) Discuss the potential long-term impacts on the labor market and overall economy.

Solution:

1. **Equilibrium Wage and Employment:**

$$\begin{aligned} L_d &= L_s \\ 50 - 2w &= 10 + 3w \\ 40 &= 5w \\ w &= 8 \end{aligned}$$

   Employment level:

$$L = 50 - 2(8) = 34$$

2. **Minimum Wage Above Equilibrium:**

$$\text{Set minimum wage } w_{\min} = 10: \quad \begin{aligned} L_d &= 50 - 2(10) = 30 \\ L_s &= 10 + 3(10) = 40 \end{aligned}$$

   Surplus:

$$L_s - L_d = 40 - 30 = 10$$

3. **Long-Term Impacts:**
   - Unemployment rises as labor supply exceeds demand.
   - Firms may invest in automation.
   - Potential long-term reduction in low-skilled job opportunities.

Exercise 9: Game Theory and Oligopoly Behavior

Problem:

1. Consider a duopoly where two firms, A and B, choose quantities to produce. The market price is determined by the total quantity produced $Q = Q_A + Q_B$ and is given by $P = 100 - Q$. Both firms have a cost function $C(Q) = 20Q$.
   (a) Derive the best response functions for both firms.
   (b) Find the Nash equilibrium quantities for both firms.
   (c) Calculate the equilibrium price and profits for each firm.
2. Discuss how the outcome would change if the firms were competing in prices (Bertrand competition) instead of quantities (Cournot competition).

Solution:

1. **Best Response Functions:**

$$\begin{aligned} \pi_A &= (100 - Q_A - Q_B)Q_A - 20Q_A \\ \frac{d\pi_A}{dQ_A} &= 100 - 2Q_A - Q_B - 20 = 0 \\ 80 - 2Q_A - Q_B &= 0 \\ Q_A &= \frac{80 - Q_B}{2} \end{aligned}$$

   Similarly for firm B:

$$Q_B = \frac{80 - Q_A}{2}$$

2. **Nash Equilibrium Quantities:** Solving the system of equations:

$$
\begin{aligned}
Q_A &= \frac{80 - \frac{80 - Q_A}{2}}{2} \\
Q_A &= \frac{80 - 40 + 0.5Q_A}{2} \\
Q_A &= \frac{40 + 0.5Q_A}{2} \\
Q_A &= \frac{40}{1.5} = 26.67 \\
Q_B &= 26.67
\end{aligned}
$$

Nash equilibrium quantities: $Q_A = Q_B = 26.67$**.**

3. **Equilibrium Price and Profits:**

$$
\begin{aligned}
\text{Total quantity:} \quad & Q = 53.34 \\
\text{Price:} \quad & P = 100 - 53.34 = 46.66 \\
\text{Profit:} \quad & \pi_A = \pi_B = (46.66 - 20) \times 26.67 = 707.11
\end{aligned}
$$

4. **Bertrand Competition:**
   - In Bertrand competition, firms compete by setting prices.
   - Assuming marginal costs are identical and constant, the equilibrium price equals marginal cost, i.e., $P = 20$.
   - Firms will keep undercutting each other's prices until price equals marginal cost.
   - Each firm produces $Q_A = Q_B = 40$ (since $100 - P = 80$ and they share the market).
   - **Profits are zero** because $P = MC$.

Exercise 10: External ities and Public Goods

Problem:

1. A factory emits pollution as a byproduct of production, affecting nearby residents. The factory's marginal cost (MC) of production is $MC = 20+Q$, where Q is the quantity produced. The marginal damage (MD) to residents is $MD = 0.5Q$. The factory's marginal benefit (MB) from production is $MB = 100 - Q$.
   (a) Determine the socially optimal level of production.
   (b) Calculate the private and social costs at this production level.
2. Discuss potential government interventions to correct the externality and evaluate their effectiveness.

Solution:

1. **Socially Optimal Production:**

$$\begin{aligned} \text{Social MC} &= MC + MD = 20 + Q + 0.5Q = 20 + 1.5Q \\ \text{Set Social MC} = \text{MB:} \quad & 20 + 1.5Q = 100 - Q \\ 2.5Q &= 80 \\ Q &= 32 \end{aligned}$$

2. **Private and Social Costs:**

$$\begin{aligned} \text{Private Cost:} \quad & MC = 20 + 32 = 52 \\ \text{Social Cost:} \quad & 20 + 1.5 \times 32 = 68 \end{aligned}$$

3. **Government Interventions:**

   - **Pigovian Tax:** Imposing a tax equal to the marginal damage ($MD = 0.5Q$).
   - **Regulation:** Setting a production cap at the socially optimal level (32 units).
   - **Tradable Permits:** Allowing firms to buy and sell the right to pollute, capping total emissions.
   - **Effectiveness:** Pigovian tax internalizes the externality; regulation ensures optimal production but may lack flexibility; tradable permits are efficient but require a well-functioning market.

Exercise 11: Intertemporal Choice and Consumption

Problem:

1. An individual has a two-period utility function $U = u(C_1) + \beta u(C_2)$, where C_1 and C_2 are consumptions in periods 1 and 2, and β is the discount factor. The individual has an initial endowment of wealth W and can save or borrow at the interest rate r.

   (a) Derive the optimal consumption path.

   (b) Analyze the impact of an increase in the interest rate on the consumption in both periods.

Solution:

1. **Optimal Consumption Path:**

$$\begin{aligned}
\text{Budget Constraint:} \quad & C_1 + \frac{C_2}{1+r} = W \\
\text{Lagrangian:} \quad & \mathcal{L} = u(C_1) + \beta u(C_2) + \lambda \left(W - C_1 - \frac{C_2}{1+r} \right) \\
\frac{\partial \mathcal{L}}{\partial C_1} &= u'(C_1) - \lambda = 0 \\
\frac{\partial \mathcal{L}}{\partial C_2} &= \beta u'(C_2) - \lambda \frac{1}{1+r} = 0 \\
\frac{u'(C_1)}{u'(C_2)} &= \beta(1+r)
\end{aligned}$$

2. **Impact of Interest Rate Increase:**

   - Higher r increases the future value of savings.
   - Substitution effect: consume less today, save more.
   - Income effect: depends on whether the individual is a net saver or borrower.

Exercise 12: Financial Markets and Investment

Problem:

1. Assume an investor has a utility function $U = E(R) - \frac{1}{2}A\sigma^2$, where $E(R)$ is the expected return, σ^2 is the variance of returns, and A is the coefficient of risk aversion.

   (a) Derive the optimal portfolio choice given a risky asset with return R_r and a risk-free asset with return R_f.

   (b) Analyze how changes in risk aversion A affect the investor's portfolio allocation.

Solution:

1. **Optimal Portfolio Choice:**

$$
\begin{aligned}
U &= E(R) - \frac{1}{2}A\sigma^2 \\
\text{Expected return:} \quad & E(R) = xR_r + (1-x)R_f \\
\text{Variance:} \quad & \sigma^2 = x^2\sigma_r^2 \\
U &= xR_r + (1-x)R_f - \frac{1}{2}Ax^2\sigma_r^2 \\
\frac{dU}{dx} &= R_r - R_f - Ax\sigma_r^2 = 0 \\
x &= \frac{R_r - R_f}{A\sigma_r^2}
\end{aligned}
$$

2. **Impact of Risk Aversion:**

   - Higher A (more risk-averse): lower allocation to the risky asset.
   - Lower A (less risk-averse): higher allocation to the risky asset.