

Chapter 4: Primitives and Integrals

1. Primitive.

- **Activity.** Let $F(x) = x^2$.
- Calculate $f(x) = F'(x)$ and conclude.
- **Definition.** It is said that a function F is an antiderivative or primitive of f on I iff, F is differentiable on I and if $\forall x \in I; F'(x) = f(x)$.
- **Example.** Give a primitive $F(x)$ of $f(x) = 3x^2$.
- **Properties.**
 1. When does a function admit a primitive?
 2. Any function continuous over an interval I admits primitives on I .
 2. The primitives of $f(x)$ are in shape $G(x) = F(x) + k; k \in \mathbb{R}$.
- Example.** $G(x) = x^2 + 2$ is a primitive also of $f(x) = 2x$.
- I.e. Each continuous function admits an infinite number of primitives

$$F'(x) = 2x = f(x)$$

$$(x^2)' = 2x$$

$$F(x) = x^3$$

$$f(x) = 2x$$

$$F(x) = x^2 + 1$$

$$x^2 + 1.5$$

$$x^2 + \sqrt{2}$$

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1. Primitives.

• Primitive table.

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$f(x)$	$F(x) = \int f(x)dx.$
0	$\int 0 dx = k; k \in \mathbb{R}$
$k; k \in \mathbb{R}$	$\int k dx = kx + c; c \in \mathbb{R}$
x	$\int x dx = \frac{x^2}{2} + k$
x^n	$\int x^n = \frac{x^{n+1}}{n+1} + k; \underline{n \neq -1}$
$\text{Si } n = -1, \quad x^{-1} = \frac{1}{x}$	$\int x^{-1} dx = \int \frac{1}{x} dx = \ln x + k$
$U' U^n \text{ where } U \text{ is a function of } x$	$\int U' U^{m+1} dx = \frac{U^{n+1}}{n+1} + k, \quad m \neq -1$
$\text{If } m = -1; \quad U' U^{-1} = \frac{U'}{U}$	$\int \frac{U'}{U} dx = \ln U + k$
e^x	$\int e^x dx = e^x + k$
$U' e^U$	$\int U' e^U dx = e^U + k$

Example: $\int x^3 dx = \frac{x^{3+1}}{3+1} + k = \frac{x^4}{4} + k \leftarrow$

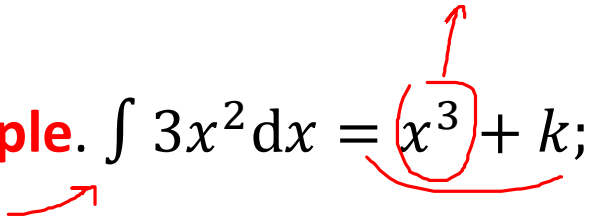
$\int 2(2x+5)^{10} dx = \frac{(2x+5)^{11}}{11} + k$
 $U = 2x+5 \rightarrow U' = 2$

$\int \frac{2}{2x+3} dx = \ln|2x+3| + k$

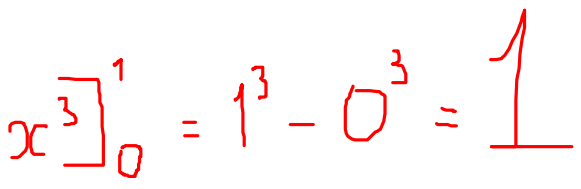
$\int 2e^{2x+1} dx = e^{2x+1} + k$

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2. Indefinite Integral.

- **Definition.** We call an indefinite integral of $f(x)$ the set of primitives $F(x) + k$. We note, $\int f(x)dx = F(x) + k$.
- **Example.** $\int 3x^2 dx = x^3 + k$; 

3. Definite integral.

- **Definition.** Let $f(x)$ be a continuous function on $[a; b]$ and $F(x)$ is a primitive of f on $[a; b]$. The definite integral of f on $[a; b]$ is given by:
- We note, $\int_a^b f(x)dx = F(x)]_a^b = F(b) - F(a)$.
- **Example.** Calculate $\int_0^1 3x^2 dx = x^3]_0^1 = 1^3 - 0^3 = 1$ 

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3. Definite integrals.

- **Properties.**

- $\int_a^b f(x)dx = -\int_b^a f(x)dx$

- $\int_a^a f(x)dx = F(a) - F(a) = 0$

- $\int_a^b kf(x)dx = k \int_a^b f(x)dx \rightarrow \int_0^2 2x^2 dx = 2 \int_0^2 x^2 dx$

- $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$
 $\int_0^2 f(x) dx = \int_0^1 f(x) dx + \int_1^2 f(x) dx$